

YBN University, Ranchi

Op-Amps, Digital electronic fundamentals, and logic gates.

Digital Electronics Fundamentals and Logic Gates.

1.1. Introduction

An electronic circuit, which processes the data or signals in the form of o's and 1's is called Digital electronics. Digital signals or digital data are the groups of o's and 1's, called binary number systems. Four major number systems are used to study and processing the digital systems or digital circuits, they are, Binary, Octal, Decimal, and Hexadecimal number systems. In the following section, the number systems and their conversions are explained.

1.2. Number systems

1.2.1. Binary Number system:

As the name implies, Bi means two, i.e., there are two symbols are used while representing the data in binary number systems. The symbols are 0 and 1. The binary number systems are also called as radix-2 system or weight -2 system.

Example: 0.1.,00,010,110101

1.2.2. Octal Number system:

A Number system with eight symbols are called Octal number systems, the symbols of this system are 0,1,2,3,4,5,6,7.The octal number systems are also called radix-8 system or weight-8 system.

Example: 012, 345, 567, 345....

1.2.3. Decimal Number System:

A Number system with Ten symbols are called Decimal number systems, the symbols of this system are 0,1,2,3,4,5,6,7,8,9. The Decimal number systems are also called radix-10 systems or weight-10 systems.

Example: 9837, 1234, 9, 3234....

1.2.4. Hexadecimal Number System:

A Number system with 16 symbols are called Hexadecimal number systems, the symbols of this system are 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F.The Hexadecimal number systems are also called radix-16 system or weight-16 system.

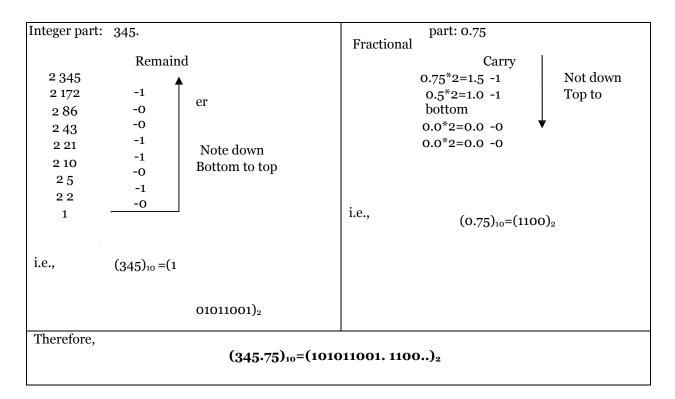
Example: 234, 01, 203A, ABF, DE4...

1.3. Number System Conversion

1.3.01. Decimal to Binary conversion

Every number has integer part and fractional part, for integer part successive division by two method and for fractional part successive multiplication by two method will be used to convert decimal number system into binary number system.

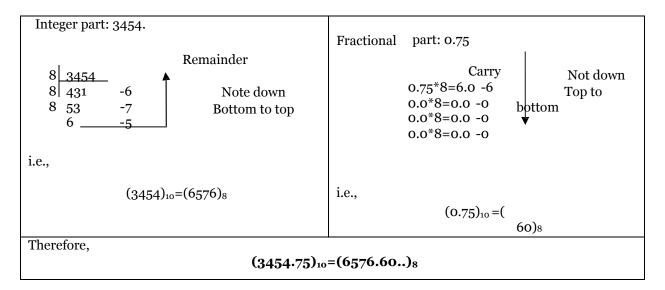
Example: (345.75)10=(?)2



1.3.02. Decimal to Octal conversion

Every number has integer part and fractional part, for integer part successive division by eight method and for fractional part successive multiplication by eight method will be used to convert decimal number system into octal number system.

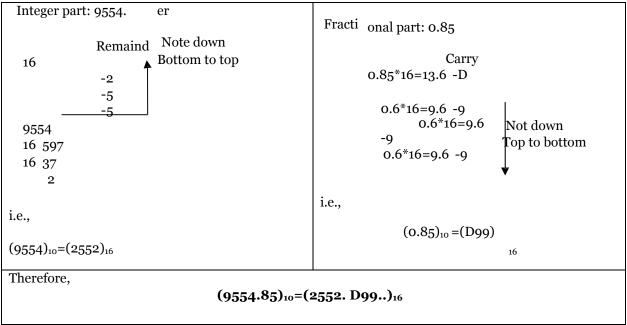
Example: (3454.75)₁₀=(?)₈



1.3.03. Decimal to Hexadecimal conversion

Every number has integer part and fractional part, for integer part successive division by 16 method and for fractional part successive multiplication by 16 method will be used to convert decimal number system into octal number system.

Example: (9554.85)₁₀=(?)₁₆



1.3.04. Binary to Decimal conversion

Multiplying sum of weighted binary numbers method is used to convert binary to decimal number system. For integer part positive weights and for fractional part negative weights will be considered.

Example: (101011001. 1100)₂=(?)₁₀

Bit position	8	7	6	5	4	3	2	1	0		-1	-2	-3
Bits	1	0	1	0	1	1	0	0	1	•	1	1	0
Weights	28	27	26	2 ₅	24	23	22	21	20		2-1	2-2	2-3
Sum	Sum = $1^{2}2^{8}+0^{27}+1^{2}2^{6}+0^{25}+1^{24}+1^{23}+0^{22}+0^{21}+1^{20}+1^{2-1}+1^{2-2}+0^{2-3}$ =345.75												
Therefore, (101011001. 1100) ₂ =(345.75) ₁₀													

1.3.05. Octal to Decimal conversion

Multiplying sum of weighted Octal numbers method is used to convert Octal to decimal number system. For integer part positive weights and for fractional part negative weights will be considered.

Example: (6576.60)₈=(?)₁₀

Bit position	3	2	1	0		-1	-2	-3
Digits	6	5	7	6	•	6	0	0
Weights	83	82	81	8 0		8-1	8-2	8-3
Sum = $6*8^{3}+5*8^{2}+7*8^{1}+6*8^{0}+6*8^{-1}+0*8^{-2}+0*8^{-3}$ = 6576.75								
Therefore, (6576.60) ₈ =(3454.75) ₁₀								

1.3.06. Hexadecimal to Decimal conversion

Multiplying sum of weighted Hexadecimal numbers method is used to convert Hexadecimal to decimal number system. For integer part positive weights and for fractional part negative weights will be considered.

Example: (2552. D99)₁₆=(?)₁₀

Bit position	3	2	1	0		-1	-2	-3
Digits	2	5	5	2	•	D	9	9
Weights	163	162	161	160		16-1	16-2	6- ₃
Sum = $2^{*16^{3}+5^{*16^{2}}+5^{*16^{1}}+2^{*16^{0}}+13^{*16^{-1}}+9^{*16^{-2}}+9^{*16^{-3}}}$ =9555.9								
Therefore, $(2552. D99)_8 = (9555.9)_{10}$								

1.3.07. Octal to Binary conversion

Replacing the binary equivalent of each octal numbers to convert octal to binary. The following table shows the binary equivalent of each octal symbol.

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Example: (67526.57)₈=(?)₂

$(110111101010110.101111)_2$

1.3.08. Hexadecimal to Binary conversion

Replacing the binary equivalent of each Hexadecimal numbers to convert Hexadecimal to binary. The following table shows the binary equivalent of each Hexadecimal symbol.

Hexadecimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
А	1010
В	1011
С	1100

D	1101
E	1110
F	1111

Example: (67AB26.5F)₁₆=(?)₂

(01100111101010100100110.0101111)₂ 1. 3.09. Binary to Octal conversion

Group the given binary number into three bits format, for integer part from right to left and for fractional part left to right. Then replace 3 bits binary to its equivalent octal. NOTE: If the number of bits are not enough to group, padding of zero's can be done.

Example: $(110111101010110).101111)_2 = (?)_8$

 $([110][111][101][010][110]).[101][111])_2 = (67526.57)_8$

1.3. 10.Binary to Hexadecimal conversion

Group the given binary number into four bits format, for integer part from right to left and for fractional part left to right. Then replace 4 bits binary to its equivalent Hexadecimal.

If the number of bits are not enough to group, padding of zero's can be done.

Example: $(01100111101010100100110.01011111)_2 = (?)_{16}$

 $([0110][0111][1010][1011][0010][0110].[0101][1111])_2 = (67AB26.5F)_{16}$

1.4. Logic gates

Logic gates are the basic building blocks of any digital systems, it is an electronic circuits, which produces one output by taking one or more inputs. The relation between output to input/s are based on certain logic, the logic operations are named as AND, OR, NOT etc..

Logic gates are classified into two types.

i. Basic Gates

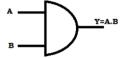
The logic gates which performs the basic operations, such as AND, OR and NOT are called basic gates.

- a. AND gate
- b. OR gate

c. NOT gate these gates are explained as follows.

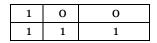
a. AND gate:

A logic gate, which performs logical AND operation is called AND gate. AND gates are having multiple input and single output. These gates produces logical value one, if all the inputs are one's else output is zero. Two inputs and single output gate is explained as follows. *Logic symbol:*



Truth table:

Inj	puts	Output
Α	В	Y=A.B
0	0	0
0	1	0



Switching Equivalent Circuit:

The working principle of AND gate can be studies using it switching equivalent circuit, is as follows.

Circuit Diagram:

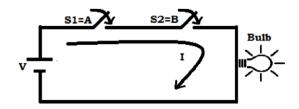


Figure: Switching Equivalent circuit

Explanation:

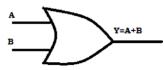
Figure shows the switching equivalent circuit of logical AND gate, which consisting of Voltage supply and series connected of two switches along with a bulb. Switches are considered as inputs A and B. case(i): If A=0 and B=0, i.e., two switches are in open state, the complete circuit is open circuit. Hence current will not flow through the circuit and energy will not be supplied to bulb, so the bulb will not glow. Hence, output Y=0.

case(ii): If A=0 and B=1, i.e., Switch A is open and Switch B is closed, the complete circuit is open circuited. hence current will not flow through the circuit and energy will not be supplied to the bulb, so the bulb will not glow. Hence, output Y=0.

case(iii): If A=1 and B=0, i.e., Switch A is Closed and Switch B is Open, the complete circuit is open circuited. hence current will not flow through the circuit and energy will not be supplied to the bulb so the bulb will not glow. Hence, output Y=0. case(iv): If A=1 and B=1, i.e., two switches are in closed state, current will flow through the circuit and energy will supply to the bulb, so the bulb will glow. Hence, output Y=1. **b. OR gate:**

A logic gate, which performs logical OR operation is called OR gate. OR gates are having multiple input and single output. These gates produces logical value one, if any one of the input is logical one, if all the input are zero's then the output is zero. Two inputs and single output gate is explained as follows.

Logic symbol:



Truth table:

Inj	puts	Output
Α	В	Y=A+B
0	0	0
0	1	1
1	0	1
1	1	1

Switching Equivalent Circuit:

The working principle of AND gate can be studies using it switching equivalent circuit, is as follows.

Circuit Diagram:

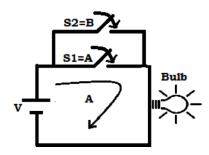


Figure: Switching Equivalent circuit

Explanation:

Figure shows the switching equivalent circuit of logical NOT gate, which consisting of Voltage supply and series connected of two switches along with a bulb. Switches are considered as inputs A and B. case(i): If A=0 and B=0, i.e., two switches are in open state, the complete circuit is open circuit. Hence current will not flow through the circuit and energy will not be supplied to bulb, so the bulb will not glow. Hence, output Y=0.

case(ii): If A=0 and B=1, i.e., Switch A is open and Switch B is closed, the circuit is closed. hence current will flow through the circuit and energy will be supplied to the bulb, so the bulb will glow. Hence, output Y=1. case(iii): If A=1 and B=0, i.e., Switch A is Closed and Switch B is Open, the circuit is closed. hence current will flow through the circuit and energy will be supplied to the bulb so the bulb will glow. Hence, output Y=1. case(iv): If A=1 and B=1, i.e., two switches are in closed state, current will flow through the circuit and energy will supply to the bulb, so the bulb will glow. Hence, output Y=1. **c. NOT gate:**

A logic gate, which performs logical NOT operation is called NOT gate. NOT gates are single input and single output gates. These gates produces logical value one, if the input is zero and output is zero if the input is one. NOT gate is explained as follows.

Logic symbol:



Truth table:

Input	Output
А	Y=A
0	1
1	0

Switching Equivalent Circuit:

The working principle of AND gate can be studies using it switching equivalent circuit, is as follows.

Circuit Diagram:

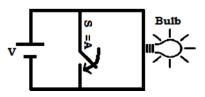


Figure: Switching Equivalent circuit

Explanation:

Figure shows the switching equivalent circuit of logical NOT gate, which consisting of Voltage supply and series connected of two switches along with a bulb. Switches are considered as inputs A and B. case(i): If A=0, i.e., Switch A is open and the circuit is closed. hence current will flow through the circuit and energy will be supplied to the bulb, so the bulb will glow. Hence, output Y=1. case(ii): If A=1 i.e., two switch A is closed, the energy will not supply to the bulb, so bulb will not glow and hence the output Y=0.

ii. Derived Gates:

The gates are designed by using the basic gates are called derived gates. There are four types of derived gates, they are. a. NAND gate

b. NOR gate

c. EXOR gate

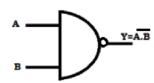
d. EXNOR gate.

The logic symbols and truth tables of all these gates are as follows.

a. NAND gate:

A logic gate, which performs logical NOT of AND operation is called NAND gate. NAND gates are having multiple input and single output. These gates produces logical value zero, if all the inputs are one's otherwise output is one. Two inputs and single output gate is explained as follows.

Logic Symbol:



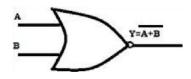
Truth Table:

Inj	puts	Output
А	В	Y=A.B
0	0	1
0	1	1
1	0	1
1	1	0

a. NOR gate:

A logic gate, which performs logical NOT of OR operation is called NOR gate. NOR gates are having multiple input and single output. These gates produces logical value one, if all the inputs are zero's otherwise output is zero. Two inputs and single output gate is explained as follows.

Logic Symbol:

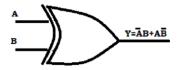


Inj	puts	Output
А	В	Y = A = B
0	0	1
0	1	0
1	0	0
1	1	0

c. EXOR Gate:

A logic gate, which performs logical Exclusive OR operation is called EXOR gate. EXOR gates are having multiple input and single output. These gates produces logical value zero, if all the inputs are o's or 1's otherwise output is one. Two inputs and single output gate is explained as follows.

Logic Symbol:



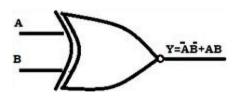
Truth Table:

Inj	outs	Output
А	В	Y=¯A+¯₿
0	0	0
0	1	1
1	0	1
1	1	0

d. EXNOR gate:

A logic gate, which performs logical Exclusive NOT of OR operation is called EXNOR gate. EXNOR gates are having multiple input and single output. These gates produces logical value one, if all the inputs are o's or 1's otherwise output is zero. Two inputs and single output gate is explained as follows.

Logic Symbol:



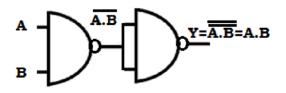
Truth Table:

Inj	puts	Output		
Α	В	Y=¯A+¯₿		
0	0	1		

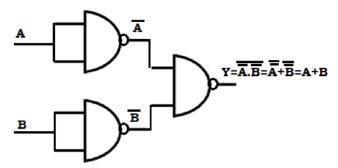
0	1	0
1	0	0
1	1 1 1	

NOTE: NAND and NOR gates are called UNIVERSAL gates, because all the basic gates can be realized by using either NAND or NOR gates only.

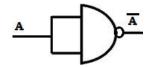
i) Realization of AND gate using NAND gates only.



ii) Realization of OR gate using NAND gates only.



iii) Realization of NOT gate using NAND gate.



1.5. Boolean Algebra

Performing algebraic operations using truth values is called Boolean algebra. The truth values are either true or false in other words binary logic levels either 0 or 1.

Boolean algebra helps to simplify the logical expressions and hence helps for designing the logical circuits.

Rules of Boolean algebra:

1. Commutative Law (a) A + B = B + A (b) A B = B A

2. Associate Law (a) (A + B) + C = A + (B + C) (b) (A B) C = A (B C)

3. Distributive Law

(a) A(B + C) = AB + AC(b) A + (B C) = (A + B) (A + C)4. Identity Law (a) A + A = A (b) AA = A5. (a) $AB + A\overline{B} = A$ (b) (A + B)(A + B) = A6. Redundance Law (a) A + A B = A (b)A(A + B) = A7. (a) 0 + A = A(b) 0 A = 08. (a) 1 + A = 1(b) 1 A = A9. $(a)A^{-}+A=1$ Α $(b)^{-}A = 0$ 10. (a) $A + \overline{A}B = A + B$ А (b) $(\overline{A} + B) = AB$ 11. De Morgan's Theorem $(a)A^{\overline{+}}B^{\overline{-}} = A \overline{-}B$ (b) $\overline{A}.\overline{B} = \overline{A} + \overline{B}$

Proof of De Morgan's laws.

Theorem-1: Complement of the sum of the two or more variables is equal to the product of the complements of their variables.

Proof:

А	В	-A	⁻В	LHSΞĀĒ₿	RHS=A B
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	0	0

Theorem-2: Complement of the product of the two or more variables is equal to the sum of the complements of their variables.

Proof:

А	В	⁻ A	⁻ B	LHS=AB	RHS=A + B
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0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

References:

1. http://www.ee.surrey.ac.uk/Projects/Labview/boolalgebra/#table2

2. https://www.tutorialspoint.com/computer logical organization/logic gates.htm

Important mathematical expressions on semiconductor diode:

1. Diode Current Equation:

$$I_F = I_o \, (e^{\frac{V}{\eta V_T}} - 1) - - - - - - - (1)$$

Where, I_F is the diode forward current, I₀ is the reverse saturation current, η is a constant, 1for Germanium and 2 for Silicon diodes, V is the voltage drop across the diode during conduction mode(Forward Biasing) and V_T is the volt equivalent Temperature given by,

$$\mathbf{V}_{\mathrm{T}} = \frac{\mathbf{k}\mathrm{T}}{\mathbf{q}} - - - - - - (2)$$

Where, k is the Boltzmann's constant (1.380649×10⁻²³J/K), T is the temperature in Kelvin and q is the charge of an electron (1.602x10⁻¹⁹C).

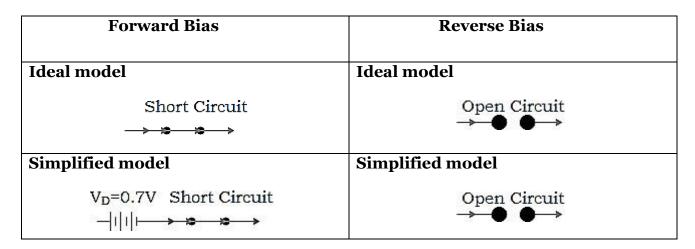
V and I₀ are temperature dependent variables.

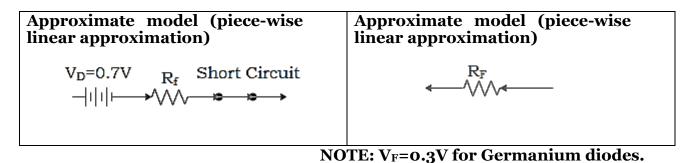
(i). I_0 doubles for every 10°C rise in temperature.

Where, I_{02} is the reverse saturation current at temperature $t_2^{\circ}C$ and I_{01} is the reverse saturation current at $t_1^{\circ}C$.

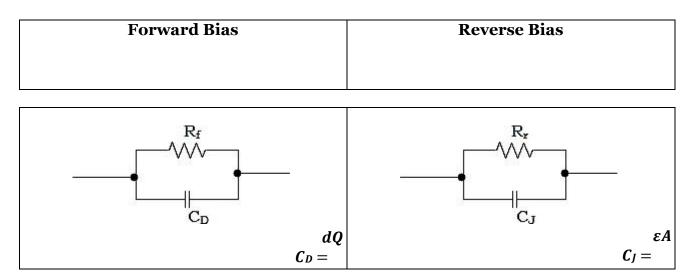
(ii). V reduces by 2.5mV for every 1°C rise in temperature.

2. Diode's DC equivalent circuits





3. Diode's AC equivalent circuits



dV W

NOTE: Dielectric Constant for Si diode is 11.9 and and for Geermanium diode is 16.0

Solved Problems

P1. Find the diode current, if the voltage across the silicon diode is 500mV at 27°C with reverse saturation current 20nA. Given Data:

V=500mV, I₀=20nA t=27°C and

 η =2(Silicon diode) To find: $\mathbf{I}_{\mathbf{F}}$

Solution:

We know that

k=1.38x10⁻²³J/K q=1.6x10⁻¹⁹C

T=27+273=300K

Therefore

 $V_T = 25.875 mV$

We know that the diode current equation

$$I_F = I_0 (e^{\frac{V}{\eta V_T}} - 1) - - - - - (1)$$

Substituting the vlue of V_T in (1),

 I_F =0.314mA or 314.11µA.

P2. A silicon diode is forward biased with voltage drop across the diode is 0.7V and Reverse saturation current is 15μ A at 26° C. Find the diode current at 34° C.

Given data:				
I01=15µA a	at t ₁ =26°C.	$V_{F1}=0.7V$	at	t1.
t ₂ =34°C ai	nd			
η =2(Silicon) To	o find:			
IF at t2				
Solution:				
We know that				
		t2-t1		
	$I_{o} = I_{o2} =$	$=> I_{o1} x 2^{10} -$		(1)
I ₀₂ =26.11	ι μΑ.			
Also,				
	$V = v_{F2} \Longrightarrow v_{F1}$	$-2.5x10^{-3}x(t_2)$	- t1) -	(2)

V_T=26.4787mV

Therefore,

$$\begin{split} I_F &= I_o \; (e^{\overline{\eta V_T}} - 1) - - - - - - - - (4) \\ I_F &= 26.11 \mathrm{x} 10^{-6} \; (e^{\frac{0.68}{2 \ast 26.47 \mathrm{x} 10^{-3}}} - 1) \\ \mathbf{I_F} &= \mathbf{9.89A} \end{split}$$

P3. A Germanium diode forward current is 20mA with voltage drop across the diode is 0.6V, find the reverse saturation current at 27°C. Given data:

IF=20mA V=0.6V

find:

Solution:

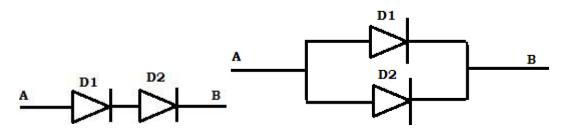
We know that,

VT=25.875mV

Therefore

I₀=1.699x10⁻¹²A

P4. Find the Voltage drop across and current through A and B for the diodes connected in i) Series and ii) Parallel shown in figures. Assume the diode currents are 25mA and voltage drop across each diode is 0.7V.



Given data:

$$I_{F1}=I_{F2}=25mA$$

 $V_1=V_2=0.7V.$

To find:

V_{AB} and I_{AB} for the above cases.

Solution:

If the two conducting diodes are connected in series, acts as a two voltage sources are connected in series,

Hence, Voltage gets added but current will be same.

If the two conducting diodes are connected in parallel, acts as two current sources in parallel. Hence, Current gets added but voltage remains same.

i.e., V_{AB}=0.7V and I_F=I_{F1}+I_{F2}=25m+25m=50mA.

P5. A silicon diode is reverse biased with a negative anode voltage of -2V, what the voltage drop is across and current through the device. Solution:

A reverse biased diode acts as an open circuit, so open circuit voltage is same as the applied voltage, i.e., **-2V** and open circuit current is **zero**.

P6. The current through the diode is 10mA at 27°C, when the applied voltage is 700mV. Find the value of η . Assume the diode current is 10⁴ times that of reveres saturation current.

Given data:

$$I_F = 10mA, t = 27 \circ C, V = 700mV \text{ and } b = \frac{I_F}{10^4}.$$

To find:

Solution:

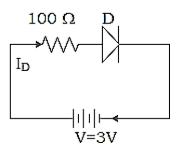
$$V_T = \frac{\kappa_I}{q}; T = 300^{\circ}K$$
$$V_T = 25.875mV$$
$$I_o = \frac{I_F}{10^4} => 0.001mA$$

η

W.k.t.,

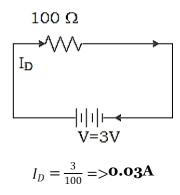
$$I_F = I_o \left(e^{\eta \overline{V_T}} - 1 \right)$$
$$\ln \left(\frac{I_F}{I_o} + 1 \right) = \frac{V}{\eta V_T}$$
$$\eta = \frac{V}{\underline{I_F}} => 2.93$$
$$\ln \left(I_o + 1 \right) > V_T$$

P7. Calculate the forward current for the diode circuit shown in figure, by assuming silicon diode with ideal, simplified and approximate model. Given forward resistance of the diode = 20 Ohms, Voltage drop across the diode=0.7V.

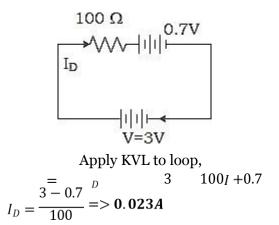


Given data: To find:

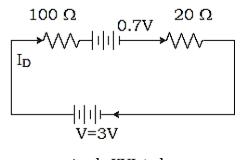
Silicon diode, V = 0.7V, $R_f = 20$ Ohms The diode forward current for ideal model, simplified model and approximate model. **Solution:** Case (i): Ideal model- Diode forward resistance is zero and acts as a short circuit, equivalent circuit is



Case (ii): Simplified model- Diode is replace by a series connection of voltage sources of 0.7V and forward resistance is zero. Equivalent circuit is



Case (iii): Approximate model- Diode is replaced by a series combination of voltage source of 0.7V and forward resistance of 20 Ohms. Equivalent circuit is



Apply KVL to loop

$$3 = 100I_D + 0.7 + 20I_D$$

 $I_D = \frac{3 - 0.7}{120} => 0.019A.$

P8. The reverse saturation current and voltage drop across the diode at 27° C is given as 10 μ A and 0.7V respectively, what is their values at 35°C? Given data:

$$I_o = 10 \mu A \text{ and } V_F = 0.7V \text{ at } t_1 = 27^{\circ}C, t_2 = 35^{\circ}C.$$

To find:

$$I_o$$
 and V_F at $t_2 = 35^{\circ}C$.

Solution: W.K.T.,

$$I'_o = I_o * 2^{\frac{t_2 - t_1}{10}}$$

 $I'_o = 17.41 \mu A$

The reverse saturation current doubles for every 100C rise in temperature and forward voltage drop across the diode decreases by 2.5mV for every 10C rise in temperature. Hence,

$$V_{F}' = V_{F} - 2.5m * (t_{2} - t_{1}) V_{F}$$

= **0**.68V

P9. A silicon diode is working under reverse biasing with an AC supply, the width of the depletion region increases to 20μ m and area has been changed to 10^{-4} m². Given relative permittivity of free space=11.9 and permittivity of free space= 8.85×10^{-12} m⁻³kg⁻¹s⁴A. Find the equivalent capacitor value. Given data:

$$W = 20 \mu m$$
, $A = 10^{-4} m^2$, $\varepsilon_r = 11.9$ and $\varepsilon_o = 8.85 x 10^{-12} M^{-3}$, $kg^{-1}S^4A$

To find:

 C_J

Solution:

We know that,

$$C_J = \frac{\varepsilon A}{W}$$
$$C_J = \mathbf{0.5265} nF$$

 $\varepsilon = \varepsilon_r \varepsilon_o$

Also,

P10. A silicon diode is working under forward biasing with an AC supply, the rate of charge carriers increases to 10nC with the applied voltage of 10V, find the equivalent capacitor value. Given data:

$$Q = 10nC$$
 and $V = 20V$

To find:

Solution:

$$C_D = \frac{dQ}{dV}$$
$$C_D = \mathbf{1}nF$$

 C_D

P11. A semiconductor diode has 10ns of reverse recovery time, find the fall time required to overcome the reverse recovery time.

Given data:

$$t_{rr} = 10ns$$

t_f

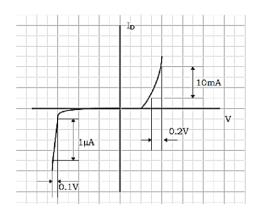
To find:

Solution:

We know that,

$$t_f \ge 10t_rr$$
$$t_f \ge 100ns.$$

P12. The VI Characteristics of a Germanium diode is shown in figure, find the equivalent forward and reverse resistance.



Given data:

$$I_F = 10 mA$$
, $I_R = 1 \mu A$, $V_F = 0.2V$ and $V_R = 0.1V$

To Find:

 R_f, R_r

Solution:

$$R_f = \frac{V_F}{\frac{I_F}{I_F}} => 20 \Omega$$
$$R_r = \frac{\frac{V_R}{I_R}}{I_R} => 10^5 \Omega$$
